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COMPUTATION OF VISUAL RANGE IN FOG AND LOW CLOUDS

Don R. Dickson

J. Vern Hales

Scientific Report #3

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Project No. 8641

Task No. 86411

December 1961

Prepared For

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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ABSTRACT

Visual ranges were computed for various values of extinction coefficient, for five sets of conditions appropriate to fog and low clouds. The computations involve basic theories of visual range as developed by Allard (1876) and Koschmieder (1924). The four values of the threshold of illumination (E_t) which Haig and Morton (1958) determined were used in the Allard theory computations.

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1. INTRODUCTION

A great need exists for better determinations of visual ranges in fog, low clouds, etc. A forward step in the effort to provide better data was made when the transmissometer AN/GMT-10 was developed.

As has been shown by Haig and Morton (1958), the Allard (1876) law proved to be very useful when used with the transmissometer data. However, an extension to the Haig and Morton work was needed for obtaining visual ranges in stratus clouds and fogs. The extension was made by use of the threshold of illumination values (E_t) of Haig and Morton along with the van de Hulst (1957) equation for the extinction coefficient (see equation 25). The drop size distributions of Bricard (1943) and Diem (1942, 1948) were used as first order approximations for fog, although these values were obtained for stratus type clouds.

2. DEVELOPMENT OF EQUATIONS

Visual range can be determined analytically by use of one of two basic assumptions. The first one which will be discussed here is the Koschmieder (1924) method. This method assumes that the object which is observed reflects scattered light incident upon it. Figure 1 illustrates this method.

Assumptions

(a) The atmosphere is considered as a turbid medium containing a large number of small particles.

(b) Each element of volume contains a large number of particles, each of small order of magnitude compared to the volume itself.

(c) The scattering action of each particle is independent of the presence of all the others.

(d) Light scattered from an element of volume will be considered as coming from a point source from which the scattered intensity is proportional to the number of particles.

(e) Light rays will be considered rectilinear; that is to say, atmospheric refraction will not be taken into consideration.

(f) A cloudless sky is assumed.

(g) In a horizontal plane near the surface of the earth, the coefficient of attenuation by scattering, b, is constant.

(h) The curvature of the earth is not considered.

(i) The linear dimensions of the object are small when

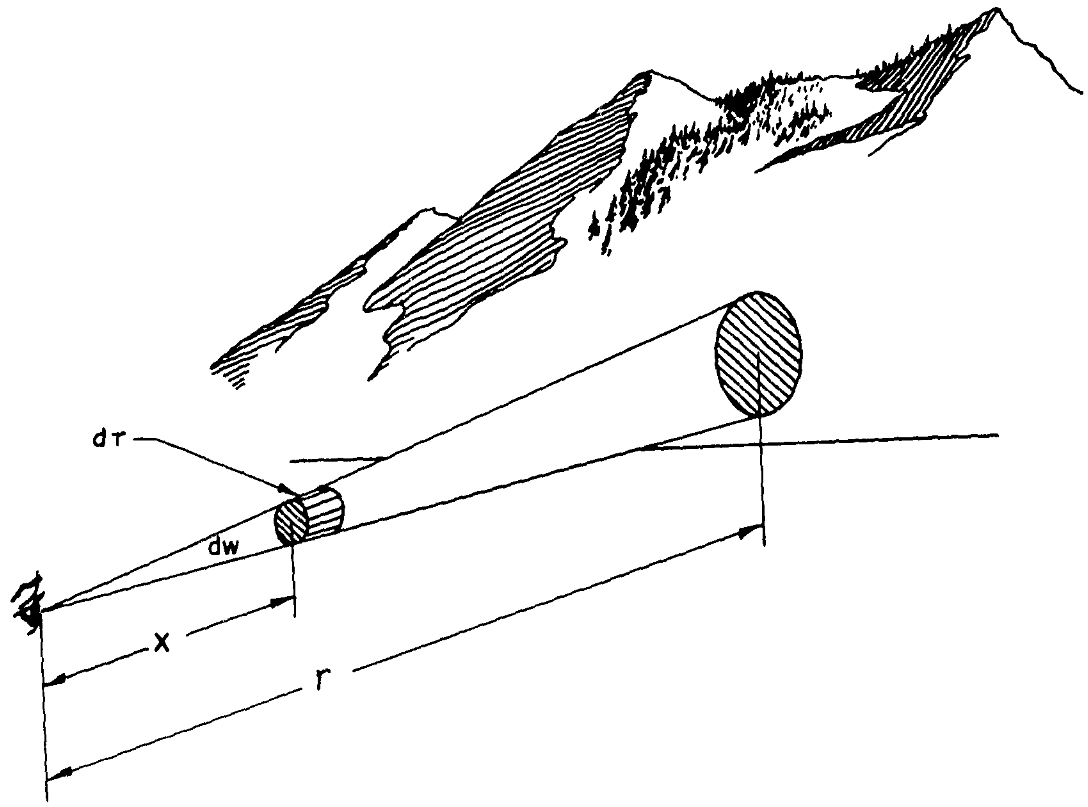

FIG. 1 ILLUSTRATING THE KOSCHMIEDER THEORY

compared to the distance from the observer.

Consider an element of volume

$$d\tau = d\omega x^2 dx, \tag{1}$$

$$dI = d\tau Ab \tag{2}$$

where A is a constant of proportionality to be determined by the boundary conditions and b (cm^{-1}) is the scattering coefficient.

The illuminance E at the eye due to the light scattered from $d\tau$ is

$$dE = dI\, x^{-2} e^{-bx} \tag{3}$$

$$dB = \frac{dE}{d\omega} \tag{4}$$

$$dB = x^{-2} e^{-bx} \frac{dI}{d\omega}\ . \tag{5}$$

Through use of equations (1) and (2), the following is obtained:

$$dB = Abe^{-bx} dx.$$

We now evaluate A by integration.

Subscript o refers to the apparent luminance of a black object, and subscript h refers to the illuminance of the horizon sky.

$$B_o = \int_0^r Abe^{-bx} dx = A(1 - e^{-br}) \tag{6}$$

$$B_h = \int_0^\infty Abe^{-bx} dx = A \tag{7}$$

therefore,

$$B_o = B_h(1 - e^{-br}). \tag{8}$$

Following Koschmieder's definition of contrast,

$$\epsilon = \frac{B_h - B_o}{B_h} \tag{9}$$

$$\epsilon = \frac{B_h - B_h(1 - e^{-br})}{B_h} = e^{-br}. \tag{10}$$

Now if r is defined as the visual range and denoted by V_m,

$$V_m = \frac{1}{b} \ln \frac{1}{\epsilon}. \tag{11}$$

Equation (11) is to be used in determining the visual range with natural daylight.

Another set of assumptions will be required for calculation of visual range when artificial lighting prevails. The Allard assumption is that the threshold of vision for a point source of light is a special case of the threshold of brightness contrast. It is more convenient to adopt as a parameter the illuminance produced at the eye by the point source. In derivation of an equation for this assumption, the total visual flux F_o emitted by a point source is attenuated by absorption and scattering. This effect can be represented by the following equation:

$$dF = \left[\left(\frac{\partial F}{\partial x}\right)_s + \left(\frac{\partial F}{\partial x}\right)_a\right] dx \tag{12}$$

$$\left(\frac{\partial F}{\partial x}\right)_s = -bF \qquad \text{Scattering} \tag{13}$$

$$\left(\frac{\partial F}{\partial x}\right)_a = -kF \qquad \text{Absorption} \tag{14}$$

the attenuation coefficient σ is given by

$$b + k = \sigma, \tag{15}$$

and the definition for illuminance E by

$$E = \frac{dF}{dx} = -\sigma F. \tag{16}$$

By integrating equation (17) between the limits 0 and r, we get

$$F = F_o e^{-\sigma r}. \tag{17}$$

Multiplying both sides of the equation by $(-\sigma)$, we get

$$-\sigma F = -\sigma F_o e^{-\sigma r} \qquad \text{and} \tag{18}$$

$$E = E_o e^{-\sigma r}. \tag{19}$$

If the light source is not a collimated beam, the intensity I_o is reduced further inversely as the square of distance r, and

$$E_t = \frac{I_o e^{-\sigma r}}{r^2} \tag{20}$$

where E_t is defined as the threshold value of E, or the minimum illuminance needed by the observer to produce the sensation of seeing.

Equation (20) is known as Allard's law when the visual range V_m is substituted for r.

$$E_t = \frac{I_o e^{-\sigma V_m}}{V_m^{\;2}} . \tag{21}$$

Equation (21) can be made more applicable by introduction of the transmissometer reading $T = e^{-\sigma R}$

$$T^{V_m/R} = e^{-\sigma V_m} . \tag{22}$$

Where R is the base line of the transmissometer, in our case 750 ft, equation (21) becomes

$$E_t = \frac{I_o T^{V_m/R}}{V_m^{\;2}} . \tag{23}$$

Solving for V_m, equation (24) is obtained.

$$2 \ln V_m + \ln E_t = \ln I_o + \frac{V_m}{R} \ln T . \tag{24}$$

The solution of equation (24) can be obtained by the use of Figures 2 and 3, a nomogram with the attenuation coefficient σ as ordinate and the visual range V_m as the abscissa.

A much more sophisticated solution can be obtained by programing equation (24) for introduction into a digital computer and feeding the transmissometer data directly into the computer. This process would allow it to be incorporated directly into the 433L System.

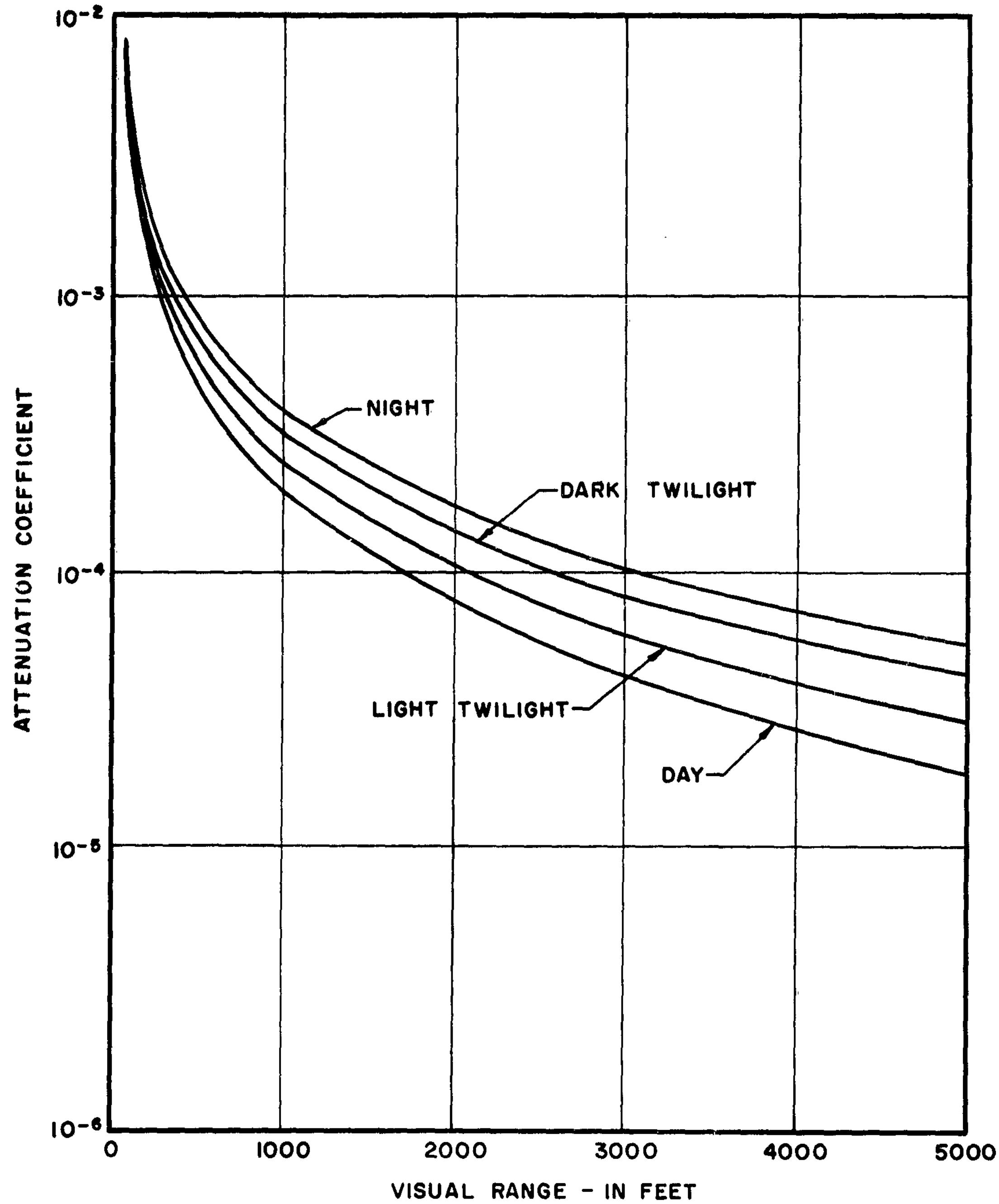

FIG. 2. THE MINIMUM DISTANCE AS A FUNCTION OF THE ATTENUATION COEFFICIENT AS SEEN BY 20 % OF THE PILOTS TESTED BY HAIG AND MORTON.

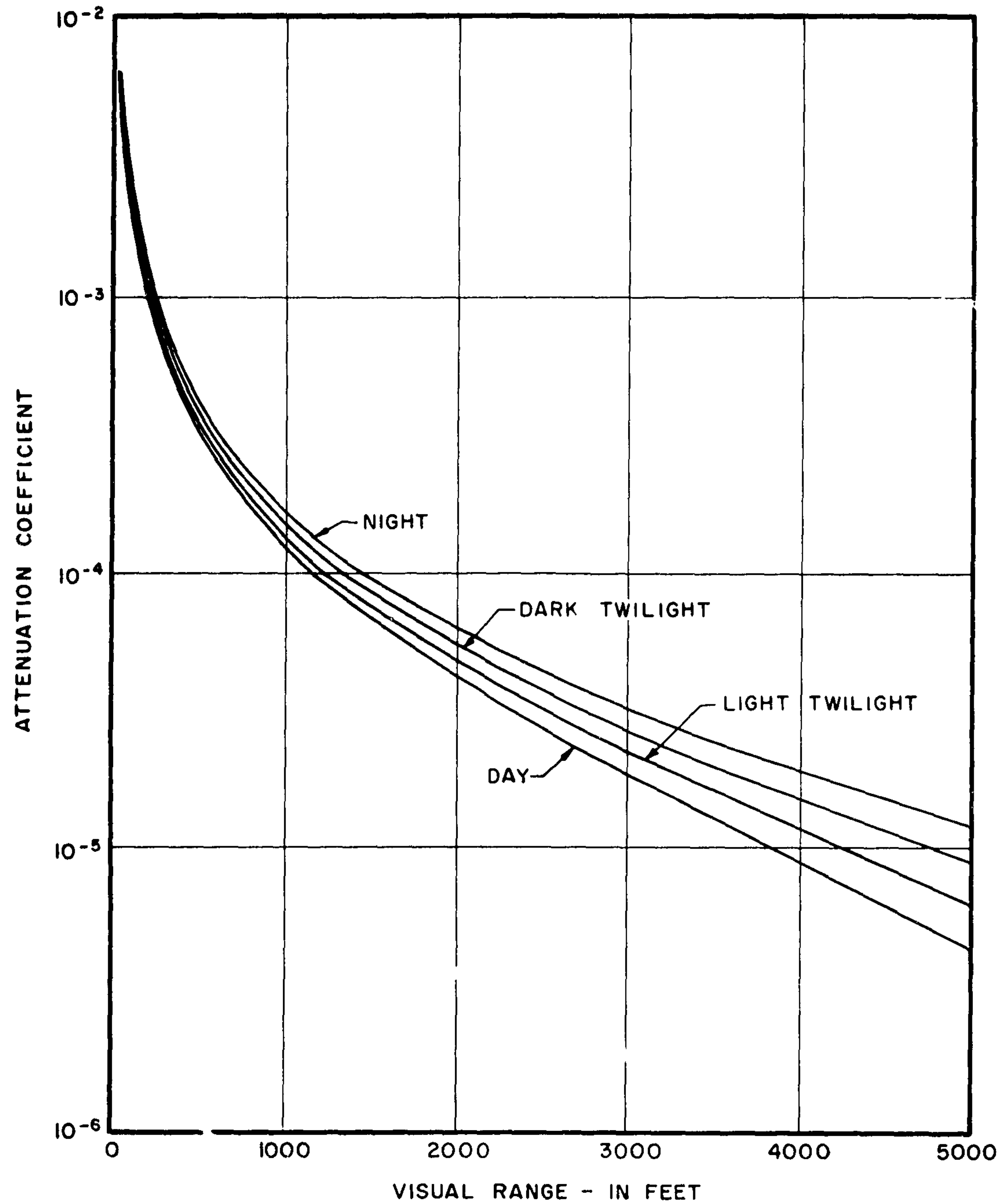

FIG. 3. THE MINIMUM DISTANCE AS A FUNCTION OF THE ATTENUATION COEFFICIENT AS SEEN BY 90% OF THE PILOTS TESTED BY HAIG AND MORTON.

3. THRESHOLD OF ILLUMINATION

Equation (24) can be used to determine the visual range when the threshold of illumination from a point source E_t is known. Haig and Morton (1958) determined what the values of E_t should be for pilots when various types of background lighting conditions prevailed. The Haig and Morton paper established the threshold of illumination at two levels, based on the individual values of E_t found for participating pilots. Level 1 was the threshold of illumination at which only 20% of all pilots tested could detect the field landing lights. Level 2 was the threshold of illumination at which 90% of all who participated in the study could detect the same lights.

The threshold of illumination was found to vary with different types of background lighting conditions. Haig and Morton differentiate the background lighting into four classes, i.e., night, dark twilight, bright twilight, and day.

TABLE 1. Threshold of illumination values (E_t) (after Haig and Morton).

Background Illuminance Values	E_t lumen/ft^2 20 Percent	90 Percent
Less than 2 lumen/ ft^2 (night)	8.5×10^{-8}	6.3×10^{-5}
2 to 100 lumen/ft^2 (dark twilight)	6.0×10^{-7}	1.0×10^{-4}
100 to 1200 lumen/ ft^2 (bright twilight)	4.8×10^{-6}	1.5×10^{-4}
1200 lumen/ft^2 or greater (day)	2.4×10^{-5}	2.1×10^{-4}

The other data needed to solve equation (24) are the length of the base line of the transmissometer and the light intensity of the transmissometer. For standard installations, these are:

$$R = 750 \text{ ft}$$

$$E_o = 10{,}000 \text{ Lumen/ft}^2.$$

At the Newark Airport installation used by Haig and Morton, the values were $R = 810$ ft, and $E_o = 10{,}000$ candlepower. Corrections for these differences have been included in our work in order to make the nomograms applicable to standard installations.

4. EXTINCTION COEFFICIENTS

In order to determine the attenuation of vision by fog and low clouds, the water droplets in suspension are all assumed to be spherical and non-absorbing in the visible region of the spectrum. For the extinction coefficient, van de Hulst (1957) gives equation (25):

$$\sigma = \pi n a^2 Q, \tag{25}$$

where n is the number of drops per unit volume, a is drop radius, and Q is defined as the efficiency factor determined by

$$Q = 2 - \frac{4}{\rho} \sin \rho + \left(\frac{2}{\rho} \right)^2 (1 - \cos \rho). \tag{26}$$

According to van de Hulst, this is one of the most useful formulae in the whole domain of the Mie theory because it describes the salient features of the extinction curve for values of index of refraction between 1 and 2. Index of refraction, m, is 4/3 for water droplets. van de Hulst defines ρ by

$$\rho = 2x \ (m - 1) \tag{27}$$

and x as

$$x = \frac{2 \pi a}{\lambda} \tag{28}$$

where λ is the predominant wave length in the visible region of the spectrum. For simplification λ was assumed to be equal to 0.5μ.

Equation (25) provides a method by which the extinction coefficients of the visual spectrum can be computed if the radius and drop concentration are known. Bricard (1943) and Diem (1942, 1948) published information concerning these parameters of stratus clouds. Bricard gave the mean radius and drop concentration, but Diem gave three characteristic drop radii and concentrations. The characteristic drop radii of Diem are defined as

r_m - the mean radius,

r_d - the mode radius,

r_p - the predominant radius.

The extinction coefficients were computed for the Bricard mean drop radius and each of the Diem characteristic drop radii. For ease in compiling the following case numbers were used:

Case			
1.	Bricard	r_m	mean drop radius
2.	Diem	r_m	mean drop radius
3.	Diem	r_d	mode drop radius
4.	Diem	r_p	predominant drop radius

Table 2 gives the needed information concerning cloud drop distributions and extinction coefficients for each of the above cases.

TABLE 2. Cloud drop size distribution, van de Hulst efficiency factor and attenuation coefficient.

Case	$\underline{n}$	$\underline{r}$	$\underline{Q}$	$\underline{\sigma}$
1.	$664 \frac{\text{drops}}{\text{cm}^3}$	5.3×10^{-4}cm	1.91977	$11.22 \times 10^{-4}\text{cm}^{-1}$
2.	$260 \frac{\text{drops}}{\text{cm}^3}$	6×10^{-4}cm	1.92269	$5.65 \times 10^{-4}\text{cm}^{-1}$
3.	$260 \frac{\text{drops}}{\text{cm}^3}$	4×10^{-4}cm	2.10139	$2.75 \times 10^{-4}\text{cm}^{-1}$
4	$260 \frac{\text{drops}}{\text{cm}^3}$	9×10^{-4}cm	2.03397	$13.46 \times 10^{-4}\text{cm}^{-1}$

5. COMPUTATION OF VISUAL RANGE

The following visual ranges were determined for each of the attenuation coefficients of Table 2. The visual ranges were computed for each of the threshold of illumination values (E_t) described by Haig and Morton (1958). These data are given in Tables 3-5 inclusive.

TABLE 3. The minimum distance that 20% of pilots tested by Haig and Morton could see.

Case	day	bright twilight	dark twilight	night
1	260 ft	298 ft	353 ft	397 ft
2	445 ft	524 ft	616 ft	717 ft
3	778 ft	930 ft	1125 ft	1329 ft
4	222 ft	260 ft	296 ft	345 ft

TABLE 4. The minimum distance that 90% of pilots tested by Haig and Morton could see.

Case	day	bright twilight	dark twilight	night
1	204 ft	210 ft	218 ft	224 ft
2	350 ft	362 ft	380 ft	400 ft
3	586 ft	615 ft	649 ft	692 ft
4	182 ft	186 ft	194 ft	202 ft

TABLE 5. The visual ranges for conditions of natural light obtained by use of the Koschmieder equation (12).

Case	V_m
1	114 ft
2	226 ft
3	465 ft
4	95 ft

The comparison between visual ranges of natural light and the visual ranges determined through use of Allard's law with the described lights for daylight conditions are given in Table 6.

TABLE 6. Visual ranges for daylight conditions, comparing Koschmieder and Allard methods.

Case	V_m-nat.	V_m-20	V_m-90
1	114 ft	260 ft	204 ft
2	226 ft	445 ft	350 ft
3	465 ft	778 ft	586 ft
4	95 ft	222 ft	182 ft

The results obtained from these formulae appear to give a very good first order approximation of the visual range in clouds and fog. From these results, an approximate method for determining the mass of water suspended in a cloud or fog formation can be developed.

The mass of cloud water per unit volume is

$$w = \frac{4}{3} \frac{\bar{a}\sigma\delta}{Q} \tag{29}$$

where δ is the density of water.

In application of this equation, the water density in the four cases examined would be shown in Table 7.

TABLE 7. Water density for the four types of clouds studied.

Case			
1	4.12×10^{-7} gm/cm^3	or	0.412 gm/m^3
2	23.50×10^{-8} gm/m^3		0.235 gm/m^3
3	6.98×10^{-8} gm/cm^3		0.0698 gm/m^3
4	7.96×10^{-7} gm/cm^3		0.796 gm/cm^3

It should be noted that Diem (1942, 1948) gives several characteristic drop parameters, and each of these parameters gives a different density of liquid water in the cloud and different visual range. This variation emphasizes that an approximation is the best which can be hoped for until new measurements become available.

There is an interesting relationship between visual range, the size of the droplets, and the water content of fogs. This relationship is even more striking when Findeisen's paper (1932) is considered. He found evidence for the coalescence of droplets in an artificial fog and stated that aging of fog is associated with a shift in drop size spectrum to larger sizes.

If this theory is correct, visual range should increase as fog ages. In order to examine the effect of the Findeisen theory, we have calculated visual range for four new cases. Each is a fog having the same liquid water content as one of those listed in Table 2, but with drop radius twice that listed in Table 2. Visual ranges calculated for the two sets of drop size distribution are given in Table 8, which shows, of course, that visual ranges are greater in fogs with the larger (and fewer) drops.

TABLE 8. The variation of visual range in an aging fog.

Case	Day		Bright Twilight		Dark Twilight		Night (1)	
	New Fog	Aged Fog	New Fog	Aged Fog	New Fog	Aged Fog	New Fog	Aged Fog
1-20	260	435	298	510	353	600	397	695
1-90	204	340	210	355	218	370	224	390
2-20	445	740	524	890	616	1070	717	1260
2-90	350	560	362	590	380	620	400	660
3-20	778	1360	930	1670	1125	2060	1329	2460
3-90	586	970	615	1030	649	1095	692	1175
4-20	222	390	260	460	296	540	345	625
4-90	182	310	186	325	194	340	202	355

(1) All visual ranges are given in feet.

6. CONCLUSION

This investigation indicated the feasibility of using a method such as has been described here in an automatic weather observing and forecasting system such as the 433L System.

The Findeisen claim that the coalescence of droplets in an artificial fog shifts the drop size spectrum to larger sizes and the Langmuir theory which says that the smaller droplets would tend to evaporate give strong support to the idea that visual ranges should increase as a fog ages.

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